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MARK SCHEME

Maximum Mark: 50

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	Take logarithms of both sides and apply power law to both sides	M1	Allow $y = \frac{\log 5}{4 \log 3}$ for M1 A1
	Rearrange to the form $y = \frac{\ln 5}{4 \ln 3} x$ or equivalent	A1	
	Obtain $m = 0.366$	A1	
	Total:	3	
2	State or imply non-modulus inequality $(4-x)^2 \leq (3-2x)^2$ or corresponding equation, pair of linear equations or linear inequalities	M1	
	Attempt solution of 3-term quadratic equation, of two linear equations or of two linear inequalities	M1	
	Obtain critical values -1 and $\frac{7}{3}$	A1	SR Allow B1 for $x \leq -1$ only or $x \geq \frac{7}{3}$ only if first M1 is not given
	State answer $x \leq -1, x \geq \frac{7}{3}$	A1	Do not accept $\frac{7}{3} \leq x \leq -1$ or $-1 \geq x \geq \frac{7}{3}$ for A1
	Total:	4	

Question	Answer	Marks	Guidance
3	Integrate to obtain form $ke^{\frac{1}{2}x+3}$ where k is constant not equal to 4	M1	
	Obtain correct $8e^{\frac{1}{2}x+3}$	A1	Allow unsimplified for A1
	Obtain $8e^{\frac{1}{2}a+3} - 8e^3 = 835$ or equivalent	A1	
	Carry out correct process to find a from equation of form $ke^{\frac{1}{2}a+3} = c$	M1	
	Obtain 3.65	A1	If 3.65 seen with no actual attempt at integration, award B1 if it is thought that trial and improvement with calculator has been used.
	Total:		5
4(i)	Use iteration correctly at least once	M1	
	Obtain final answer 2.08	A1	
	Show sufficient iterations to 4 dp to justify answer or show sign change in interval (2.075, 2.085)	A1	
	Total:		3
4(ii)	State or clearly imply equation $x = \frac{2x^2 + x + 9}{(x+1)^2}$ or same equation using α	B1	
	Carry out relevant simplification	M1	
	Obtain $\sqrt[3]{9}$	A1	
	Total:		3

Question	Answer	Marks	Guidance
5(i)	State $R = 3$	B1	Allow marks for (i) if seen in (ii)
	Use appropriate trigonometric formula to find α	M1	
	Obtain 48.19 with no errors seen	A1	
	Total:	3	
5(ii)	Carry out evaluation of $\cos^{-1}\frac{1}{3}$ ($= 70.528\dots$)	M1	M1 for $\cos^{-1}\left(\frac{1}{R}\right)$
	Obtain correct answer 118.7	A1	
	Carry out correct method to find second answer	M1	
	Obtain 337.7 and no others between 0 and 360	A1	
	Total:	4	
6(i)	State or imply correct y-values $0, \tan\frac{1}{6}\pi, \tan\frac{2}{6}\pi$	B1	Some candidates have their calculator in degree mode when working out $\tan\frac{\pi}{6}$ etc. this gives 0.00915 and 0.0183. Allow B1 .
	Use correct formula, or equivalent, with $h = \frac{1}{12}\pi$ and y-values	M1	Must be convinced they have considered 3 values for y for M1
	Obtain 0.378	A1	
	Total:	3	

Question	Answer	Marks	Guidance
6(ii)	State or imply $\pi \int (\sec^2 2x - 1) dx$	B1	
	Integrate to obtain $k_1 \tan 2x + k_2 x$, any non-zero constants including π or not	M1	
	Obtain $\frac{1}{2} \tan 2x - x$ or $\pi \left(\frac{1}{2} \tan 2x - x \right)$	A1	
	Obtain $\pi \left(\frac{1}{2} \sqrt{3} - \frac{1}{6} \pi \right)$ or equivalent	A1	
	Total:	4	
7(i)	Differentiate x and y and form $\frac{dy}{dx}$	M1	
	Obtain $\frac{4t^3 - 6t^2 + 8t - 12}{3t^2 + 6}$	A1	First 2 marks may be implied by an attempt at division
	Carry out division at least as far as kt or equivalent	M1	For M1 , it must be division by a quadratic factor. Allow attempt at factorisation with same conditions as for division
	Obtain $\frac{4}{3}t$	A1	
	Obtain $\frac{4}{3}t - 2$ with complete division shown and no errors seen	A1	
	Total:	5	

Question	Answer	Marks	Guidance
7(ii)	State or imply gradient of straight line is $\frac{1}{2}$	B1	Allow B1 if $y = \frac{1}{2}x + \frac{9}{2}$ is seen
	Attempt value of t from their $\frac{dy}{dx} =$ their negative reciprocal of gradient of line	M1	
	Obtain $t = 0$ and hence $(1,5)$	A1	
	Total:	3	
8(i)	Apply product rule to find first derivative	*M1	
	Obtain $6x \ln\left(\frac{1}{6}x\right) + 3x$ or equivalent	A1	Allow unsimplified for A1
	Identify $x = 6$ at P	B1	
	Substitute their value of x at P into attempt at first derivative	DM1	dep *M
	Obtain 18	A1	
	Total:	5	

Question	Answer	Marks	Guidance
8(ii)	Equate their first derivative to zero and attempt solution of equation of form $k \ln\left(\frac{1}{6}x\right) + m = 0$	*M1	
	Obtain x -coordinate of form $a_1 e^{a_2}$	DM1	dep *M
	Obtain $x = 6e^{-\frac{1}{2}}$ or exact equivalent	A1	
	Substitute exact x -value in the form $a_1 e^{a_2}$ and attempt simplification to remove \ln	M1	
	Obtain $-54e^{-1}$ or exact equivalent	A1	
	Total:	5	